Effect of Variable Thermal Conductivity on Natural Convection and Radiation in Porous Fins

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Abstract- In this study, the effects of variable thermal conductivity on radiation and convection heat transfer in porous media are considered. The geometry considered is that of a rectangular profile fin. The thermal conductivity is assumed to be a function of temperature. To study the thermal performance, three types of cases are considered viz. long fin, finite length fin with insulated tip and finite length fin with tip exposed. The effects of the porosity parameter \( S_h \), radiation parameter \( G \), temperature ratio \( C_T \) and the thermal conductivity parameter \( m \) on the dimensionless temperature distribution and heat transfer rate are discussed. The results suggest that the thermal conductivity parameter has a lot of influence on heat transfer rates.

Keywords- Porous Fins; Natural Convection; Thermal Analysis; Variable Thermal Conductivity

Nomenclature

\begin{itemize}
  \item Bi Biot Number, \( \frac{k_b}{k_f} \)
  \item \( C_p \) Specific heat
  \item \( C_T \) Temperature ratio \( \frac{T_b - T_s}{T_b - T_\infty} \)
  \item Da Darcy number, \( K/t^{2} \)
  \item \( g \) gravity constant
  \item \( G \) Radiation parameter
  \item \( Gr \) Grashof number
  \item \( k \) Thermal conductivity.
  \item \( K_r \) Thermal conductivity ratio \( \frac{k_{eff}}{k_f} \)
  \item \( K \) Permeability of porous fin
  \item \( L \) Length of the fin
  \item Nu Nusselt number, \( \frac{hL}{k_f} \)
  \item Pr Prandtl number, \( \frac{C_p k}{\nu} \)
  \item \( q \) Heat transfer rate
  \item \( q_r \) Radiation heat transfer rate
  \item \( Ra \) Rayleigh number, \( (Gr . Pr) \)
  \item \( S_h \) Porous parameter
  \item \( T(x) \) Temperature at any point
  \item \( T_b \) Temperature at fin base
  \item \( t \) Thickness of the fin
  \item \( u(x) \) Velocity of fluid passing through the fin any point.
  \item \( W \) Width of the fin
  \item \( x \) Axial coordinate
\end{itemize}

Greek Symbols

\begin{itemize}
  \item \( \alpha \) Thermal diffusivity
  \item \( \beta \) Coefficient of volumetric thermal expansion
  \item \( \lambda \) thermal conductivity parameter
  \item \( \Delta \) Temperature difference
  \item \( \varepsilon \) Porosity or void ratio
  \item \( \sigma \) Stephen-Boltzmann constant, \( \sigma = 5.6703 \times 10^{-8} \text{ (W/m}^2\text{K}^4) \)
  \item \( \varepsilon \) Emissivity of porous fin
  \item \( \theta \) Dimensionless temperature, \( \theta = \frac{T-T_s}{T_b-T_s} \)
  \item \( \theta_b \) Base temperature difference, \( (T_b - T_\infty) \)
  \item \( \nu \) Kinematic viscosity
  \item \( \rho \) Density
\end{itemize}

Subscripts

\begin{itemize}
  \item \( b \) base of the fin
  \item \( s \) Solid properties
  \item \( f \) Fluid properties
  \item \( eff \) Porous properties
\end{itemize}

I. INTRODUCTION

As high-power electronics continue to push power density limits, component design engineers are facing greater challenges and trade-offs in their choice for cooling solutions. One way to meet these challenges and trade-offs is through the engineering of fin geometry and fin density of heat transfer devices such as heat exchangers. This paper will explain how fin geometry affects the performance of heat exchangers. It will briefly review some basic heat transfer theory and the role of fins in improving performance, and focus on minimizing thermal resistance as a way to maximize performance.

A thermoelectric module essentially consists in a solid state device operating as a heat pump on the basis of the Peltier effect. A thermoelectric module typically comprises a plurality of pairs of elements made of semiconductor material of p and n type, the so called "thermoelectric elements", which are connected electrically in series and
thermically in parallel and arranged between two plates of ceramic material serving as a substrate for the thermoelectric elements. When an electric current is passed through the thermoelectric elements, heat is transferred from one side to the other of the thermoelectric module where it is removed by a heat sink. Generally, the heat sink is of the air cooled type and has a finned construction. Various methods are known in the art for making a heat sink having a finned structure. According to a first known method, the finning of the heat sink is obtained by extrusion from a single piece of material and is made integrally with a base portion of the heat sink which has a flat surface intended to be connected to the hot side of the thermoelectric module, i.e. the side from which heat has to be removed. This method of manufacturing the heat sink has a low cost, but has a limit in that it is difficult to make a great number of long fins with narrow gaps between them. In a second known method, the finning of the heat sink is made from a base portion of the heat sink having a flat surface intended to be connected to the hot side of the thermoelectric module and a series of grooves having a shape so that the fins can be inserted therein by applying a pressure, thus forming a group of parallel fins. This method of manufacturing the heat sink has a high cost because of the difficulty of assembling a great number of fins separated by a small gap on the base portion and because of the need to have recourse mainly to hand manufacturing for connecting the fins to the base portion of the heat sink, which connection has to be made by an interference fit in order to achieve a good contact between the base and the fins. Heat transfer through fins finds applications in several engineering systems requiring dissipation of heat \[^1\-\^{15}\]. The compact size and cost of the fin with high heat transfer rate are very important parameters to be considered while using fins. To reduce the operating cost, it should have smaller weight. As a result, enhancement of heat transfer is the focus of current research. Three types of heat transfer take place in a fin namely, conduction, convection and radiation. One achieves maximum fin efficiency by increasing heat transfer rate in the fin. Convection and conduction heat transfer are main heat transferring processes considered but radiation heat transfer has a significant effect on cooling fin while transferring maximum heat from the fin to the surrounding fluid.

Kiwan and Al-Nmr \[^1\] analyzed natural convection in porous fins. Nguyen and Aziz \[^3\] compared the heat transfer rates from convecting-radiating non-porous fins for different profile shapes. Mueller and Abu-Mulaweh \[^3\] evaluated the efficiency of horizontal single non-porous fin subjected to free convection and radiation heat transfer. Kang and Look \[^4\] and Razelos and Kakatsios \[^5\] presented optimum designs of a thermally asymmetric convecting and radiating non-porous rectangular annular fin. Yu and Chen \[^6\] performed a study on optimization of circular non-porous fins with variable thermal parameters. Gorla and Bakier \[^7\] recently investigated the effect of radiation on the performance of porous fins in a natural convection environment.

Acharya and Patankar \[^8\] investigated the effect of buoyancy on laminar mixed convection in a shrouded fin array. Harahap and McManus \[^9\] conducted an experimental investigation on natural convection heat transfer from horizontal fin arrays. They found that the heat transfer coefficients for horizontal fin channels are lower than the heat transfer coefficients for vertical fin channels. Van de Pol and Tierney \[^10\] concluded from their study that the vertical base plate/vertical fin channel configuration shows the best performance for natural convection cooling. Elenbaas \[^11\] considered heat dissipation of parallel plates by natural convection. Prakash and Lounsbury \[^12\] analyzed the problem of laminar fully developed flow in finned parallel plate passage. They considered number of cross-sectional shapes of the fin and studied the effect of fin shape on various system parameters. Lakhal et al. \[^13\] studied the natural convection heat transfer in inclined enclosures with perfectly conducting fins attached to the heated wall. They reported that heat transfer through the cover is considerably affected by the presence of the fins. Rao et al. \[^14\] developed a numerical model treating the adjacent internal fins as two-fin enclosures for study the heat transfer from horizontal fin array. They studied the effects of system parameters like fin height, fin spacing, base temperature and emissivity on rate of heat transfer from fin array. Rao and Venkateshan \[^15\] conducted experiments on horizontal fin arrays. They studied the effects of fin height, fin spacing, fin array base temperature and, fin emissivity on heat transfer rates from fin array.

The present work has been undertaken in order to extend the work done by Gorla and Bakier \[^7\] and the investigate the effect of variable thermal conductivity on the performance of porous fins in a natural convection and radiation environment. The effective thermal conductivity is assumed to be a function of temperature. The effect of the porous parameter \(\lambda\), radiation parameter \(G\), temperature ratio \(C_T\) and thermal conductivity parameter \(m\) on heat transfer rate and temperature distribution have been determined.

II. MATHEMATICAL MODEL AND ASSUMPTIONS

As shown in Fig. 1, a rectangular fin profile is considered. The dimensions of the fin are length \(L\), width \(W\) and thickness \(t\). The cross section area of the fin is constant. This fin is porous and allows the flow of infiltrate through it. The following assumptions are made to solve this problem \[^7\].
1. The porous medium is isotropic and homogeneous.
2. The porous medium with pores completely filled with a single-phase fluid.
3. The surface radiant exchange is neglected.
4. Thermal conductivity of the fluid is a function of temperature.
5. The temperature inside fin is only function of $x$.
6. The solid matrix and fluid are assumed to be at local thermal equilibrium with each other.

The interactions between the porous medium and the clear fluid can be simulated by the Darcy formulation. From the Darcy’s model, 

\[ \nu_B \sim \frac{\rho \mathcal{E}}{\nu (\tau - \tau_w)} \]

A. Formulation

The equation governing the temperature distribution in the fin is based on an energy balance between the heat conducted in the fin, surface radiation and the energy convected out by the infiltrate. Assume variable thermal conductivity is

\[ k = k_0 \left(1 + \lambda (T - T_{in})\right). \]

Then the dimensionless version of this equation is given by:

\[ \frac{d^2 \theta}{dx^2} - \frac{S_b \theta^2}{1 + m \theta} - C_T \left( \frac{\theta}{1 + m \theta} \right)^4 - C_T^4 = 0 \]  

\[ (1) \]

where

\[ \theta = \frac{T - T_{in}}{T_b - T_{in}}, \quad m = \frac{T_b - T_{in}}{T_b - T_{in}}, \quad C_T = \frac{T_{in}}{T_b - T_{in}}, \quad X = \frac{x}{L} \]

\[ S_b = \frac{D_A R_B (L)}{t}; \quad \text{Porous parameter}, \]

\[ G = \frac{2 \sigma m L^2 (\Delta T)^2}; \quad \text{Radiation parameter}. \]

The heat flux will be

\[ q = -k_0 A_b \left( T_{in} - T_{in} \right) \left(1 + m \right) \left( \frac{d \theta}{dx} \right) \]

\[ \theta = \frac{0}{1 + m \theta} \]

numerically using the spectral collocation method \cite{16, 17}. Depending on the tip condition of the fin, we have three different types of cases:

A. Long Fin

For this case, the fin tip temperature will be almost equal to temperature of surrounding fluid.

Figure 2 shows the variation of dimensionless temperature distribution with the axial distance along the fin when the value of $S_b$ is varying and values of $G$, $C_T$ and $m$ were kept constant. We observe that the value of dimensionless temperature decreases along the fin length. As the value of $S_b$ increases, the temperature decreases rapidly and the fin quickly reaches the surrounding temperature. As the value of $S_b$ increases, the fins cool down rapidly. This is to be expected because as the value of $S_b$ increases, the permeability of the porous media and/or buoyancy effects increase and these in turn will cause increased mass flow rate and cooling effect.

![Fig. 2 The distribution of axial non-dimensional temperature along the infinitely long fin for $C_T = 0.4$, $m = 0.5$ and $G = 0.1$ for different values of $S_b$.](image)
Figure 3 displays results for the effect of variation of $C_T$ on dimensionless temperature distribution. As $C_T$ increases the temperature of a given axial location decreases. $C_T$ is the ratio of the ambient temperature to the excess temperature of the base of the fin. As this ratio increases, the base temperature becomes relatively lower and therefore the fin is cooler.

![Figure 3 - Distribution of axial non-dimensional temperature along the infinitely long fin for different values of $C_T$.](image)

Fig. 3 The distribution of axial non-dimensional temperature along the infinitely long fin for $S_h = 1$, $m = 0.1$ and $G = 1$ for different values of $C_T$.

Figure 4 indicates the effect of dimensionless radiation parameter $G$ on temperature distribution. As the value of $G$ increases from 0.1 to 1, the temperature of the fin decreases at a given axial location. As $G$ increases, the surface emissivity of the fin increases and therefore the radiation heat transfer is enhanced. This produces more cooling effect along the length of the fin.

![Figure 4 - Distribution of axial non-dimensional temperature along the infinitely long fin for different values of $G$.](image)

Fig. 4 The distribution of axial non-dimensional temperature along the infinitely long fin for $S_h = 1$, $m = 0.1$ and $G = 1$ for different values of $G$.

Figure 5 illustrates results for temperature distribution as the thermal conductivity parameter, $m$ varies. We observe that as $m$ increases, the temperature of the fin increases. As the value of $m$ increases, the base temperature becomes relatively higher and therefore the fin is hotter.

![Figure 5 - Distribution of axial non-dimensional temperature along the infinitely long fin for different values of $m$.](image)

Fig. 5 The distribution of axial non-dimensional temperature along the infinitely long fin for $S_h = 1$, $G = 1$ and $C_T = 0.2$ for different values of $m$.

**B. Finite Length Fin with Insulated Tip**

For this case, the fin tip is insulated so that there will not be any heat transfer at the insulated tip.

Figure 6 shows variation of temperature with distance of the fin while $S_h$ was varied from 1 to 100. As the value of $S_h$ increases, there is a rapid decrease in the fin temperature at a given axial location. Hence, as the values of $S_h$ increases, the fin cools down faster and quickly reaches the surrounding temperature. As the value of $S_h$ increases, the permeability of the porous media and/or buoyancy effects increase and these in turn will cause increased mass flow rate and cooling effect.

![Figure 6 - Effect of porous parameter $S_h$ on the distribution of axial non-dimensional temperature along the finite fin with an insulated tip for $C_T = 0.6$, $m = 1$ and $G = 1$.](image)

Fig. 6 Effect of porous parameter $S_h$ on the distribution of axial non-dimensional temperature along the finite fin with an insulated tip for $C_T = 0.6$, $m = 1$ and $G = 1$.

Figure 7 explains the effect of variation of $C_T$ on the temperature distribution. As $C_T$ increases, the temperature distribution decrease rapidly and fin cools down faster. As this ratio increases, the base temperature becomes relatively lower and therefore the fin is cooler.

![Figure 7 - Distribution of axial non-dimensional temperature along the infinitely long fin for different values of $C_T$.](image)

Fig. 7 The distribution of axial non-dimensional temperature along the infinitely long fin for $S_h = 1$, $G = 1$ and $C_T = 0.2$ for different values of $C_T$. 

Figure 7 elucidates the effect of the radiation parameter \( G \) on axial temperature along the length of fin for insulated tip condition at the end. As the value of \( G \) increases, the temperature at a given axial location decreases. As \( G \) increases, the surface emissivity of the fin increases and therefore the radiation heat transfer is enhanced. This produces more cooling effect along the length of the fin.

Figure 8 displays the effect of the thermal conductivity parameter \( \lambda \) on axial temperature along the length of fin for insulated tip condition at the end. As the value of \( \lambda \) increases, the temperature at a given axial location increases. As the value of \( \lambda \) increases, the base temperature becomes relatively higher and therefore the fin is hotter.

C. Finite Length Fin with Known Convective Coefficient at the Tip

For this case, neither temperature difference nor temperature gradient between fin and surrounding fluid is zero at the base of fin and the at the fin tip. Figure 10 illustrates temperature distribution along the length of fin for variation of Biot number. As Biot number increases, the temperature decreases. This implies rapid cooling as Biot number increases. Increasing values of Bi imply enhanced heat transfer coefficient and heat transfer rates from the fin, thus causing a rapid cooling effect.

Figure 11 displays results for temperature gradient along the fin length of fin for different Biot number values. The temperature gradient decreases along the length of fin for all Biot numbers. As Biot number increases, the heat transfer rate along the length of the fin increases. This is expected because of the enhanced heat transfer coefficient.
Figures 12-14 show results for temperature gradient along the fin length of fin for different values of the thermal conductivity parameter, \( m \) for Cases 1, 2 and 3 respectively. As \( m \) increases, the heat transfer rate decreases. This factor should help the designer of cooling systems. As the value of \( m \) increases, the base temperature becomes relatively higher and therefore the fin is hotter.

### IV. CONCLUDING REMARKS

Thermal analysis of a porous fin for natural convection and radiation heat transfer has been performed here. The thermal conductivity was assumed to be a function of temperature. The governing parameters influencing the temperature distribution have been grouped as \( S_n \), \( G \), \( C_r \) and \( m \). The thermal analysis was performed on three types of fin cases: the infinite fin, finite length fin with insulated tip and finite length fin with known convective coefficient at the tip. As the thermal conductivity parameter, \( m \) increases, the heat transfer rates decrease.

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### REFERENCES

